University of Palestine
Faculty of Applied Engineering and Urban Planning

Course Name	Discrete Mathematics			Course No.	SWEN 2308
Academic Year	2018/2019	Semester	2nd	Exam Type	Final
Exam Date	29/05/2019		Exam Time	9am – 11am	

الرقم الجامعي:			اسم الطالب(بالعربي):
الرقم المتسلسل:	وقت المحاضرة:	رقم الشعبة:	اسم المحاضر:
	١٢:٠٠-١١:٠٠	١٠١	د.م. أحمد حمدي أبو عبسة
	١١:٠٠-٩:٣٠	١٠٢	

Important Instructions

1. This exam consists of **eight** pages, including this page, containing **five** questions.
2. You have to answer all **five** questions.
3. The exam is closed book and closed notes. Non-programmable calculators are allowed. Make sure you turn off your mobile phone and keep it in your pocket if you have one.
4. The questions are **equally weighed**.
5. This exam is out of **50** points.
6. You have exactly **120** minutes to finish the exam.
7. Make sure your answers are **readable**.
8. If there is no space on the front of the page, feel free to use the back of the page. Make sure you indicate this in order for me not to miss grading it.

For Teacher's Use Only

QN	BN	SO	DL	Mark	Weight
1	B1	a, e and h	2		2
	B2	a, e and h	2		2
	B3	a, e and h	2		2
	B4	a, e and h	3		4
2	B1	a, e and h	2		4
	B2	a, e and h	3		6
3	B1	a, e and h	4		3
	B2	a, e and h	4		7
4	B1	a, e and h	4		10
5	B1	a, e and h	5		10
Total					**50**

For Proctor's Remarks

KPI: Key Performance Indicator, ILO: Intended Learning Outcomes, SO: ABET Student Objectives, DL: Difficulty Level (1. Very easy, 2. Easy, 3. Moderate, 4. Somewhat hard, 5. Hard, 6. Very Hard)

Q1: A. Propositional logic. ***(2 marks)***

For each of these sentences, determine whether an inclusive or, an exclusive or, is intended. Explain your answer.

a) Experience with C++ or Java is required. *(1 mark)*

b) To enter the country you need a passport or a voter registration card. *(1 mark)*

B. Logical equivalences ***(2 marks)***

Evaluate each of these expressions.

a) 1 1000 ∧ (0 1011 ∨ 1 1011) *(1 mark)*

b) (0 1010 ⊕ 1 1011) ⊕ 0 1000 *(1 mark)*

C. Predicates and quantifiers ***(2 marks)***

Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase “It is not the case that.”)

a) All dogs have fleas. *(1 mark)*

b) Every koala can climb. *(1 mark)*

D. Rules of inference ***(4 marks)***

D.1 Express the negations of each of these statements so that all negation symbols immediately precede predicates.

a) $\exists z \forall y \forall x T(x, y, z)$ *(1 mark)*

b) $\exists x \exists y (Q(x, y) \leftrightarrow Q(y, x))$ *(1 mark)*

D.2 Use predicates, quantifiers, logical connectives, and mathematical operators to express the statement that there is a positive integer that is not the sum of three squares.
(2 marks)

Q2 A. Sets, functions *(4 marks)*

A.1 Suppose that the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Express each of these sets with bit strings where the ith bit in the string is 1 if i is in the set and 0 otherwise. *(2 marks)*

a) $\{3, 4, 5\}$ *(1 mark)*

b) $\{2, 3, 4, 7, 8, 9\}$ *(1 mark)*

A.2 Draw the Venn diagrams for the combinations of the sets A, B, and C. *(2 marks)*
$(A - B) \cup (A - C) \cup (B - C)$

B. Sequences and Linear Recurrence Relations *(6 marks)*

For the sequence is defined below:

$a_n = a_{n-1} - a_{n-2}, a_0 = 2, a_1 = -1$

a) Find the first three terms of the sequence defined by the above recurrence relation and initial conditions. *(2 marks)*

b) Solve the recurrence relation *(4 marks)*

Q3. A. Proofs ***(3 marks)***

Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even. *(3 marks)*

B. **Counting, pigeonhole** ***(7 marks)***

B.1 How many strings of eight uppercase English letters are there: *(3 marks)*

a) if letters can be repeated? *(1 mark)*

b) that start with X, if letters can be repeated? *(1 mark)*

c) that start and end with the letters BO (in that order), if letters can be repeated? *(1 mark)*

B.2 A computer network consists of six computers. Each computer is directly connected to at least one of the other computers. Show that there are at least two computers in the network that are directly connected to the same number of other computers. *(2 marks)*

B.3 What is the coefficient of x^7 in $(1 + x)^{11}$? *(2 marks)*

Q4. Relations *(10 marks)*

Q4.1 How many different relations are there from a set with m elements to a set with n elements? *(1 mark)*

Q4.2 Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if: *(9 marks)*
a) $x + y = 0$. *(3 marks)*

b) $x - y$ is a rational number. *(3 marks)*

c) $x = 1$. *(3 marks)*

Q5: Choose the correct answer from the following choices. Note that the number of choices is not equal for all questions. ***(10 marks)***

1. The negation of the proposition: "Ahmad and Salim are present" is
   (a) Ahmad and Salim are absent.
   (b) Ahmad or Salim is absent.
   (c) Ahmad is present and Salim is absent.
   (d) Ahmad is absent and Salim is present
   (e) all of the above.

2. On the island of knights and knaves you encounter two people. *A* and *B*. Person *A* says, "*B* is a knave." Person *B* says, "At least one of us is a knight." Then,
   (a) *A* is a knave, *B* is a knave.
   (b) *A* is a knight, *B* is a knight.
   (c) *A* is a knight, *B* is a knave.
   (d) *A* is a knave, *B* is a knight.
   (e) one of them cannot be determined for sure whether he is knave or knight.

3. Suppose the variable *x* represents students and *y* represents courses, and:
   $U(y)$: *y* is an upper-level course $M(y)$: *y* is a math course
   $F(x)$: *x* is a freshman $A(x)$: *x* is a part-time student
   $T(x, y)$: student *x* is taking course *y*.
   Then, the statement "Every part-time freshman is taking some upper-level course" is formulated by
   (a) $\forall x \exists y[U(y) \wedge T(x,y)]$.
   (b) $\exists y \forall x[U(y) \wedge T(x,y)]$.
   (c) $\forall x \exists y[(F(x) \wedge A(x)) \rightarrow (U(y) \wedge T(x,y))]$.
   (d) $\exists y \forall x[(F(x) \wedge A(x)) \rightarrow (U(y) \wedge T(x,y))]$.
   (e) more than one answer above.

4. $A - (B \cap C)$ is equivalent to
   (a) $(A - C) \cup (A - B)$.
   (b) $(A - B) \cap (A - C)$.
   (c) $(A - B) \cup \mathrm{C}$.
   (d) $A \cup (B - C)$.
   (e) more than one answer above.

5. Suppose $A = \{x,y\}$ and $B = \{x,\{x\}\}$. Then, it is true that
   (a) $x \subseteq B$.
   (b) $\varnothing \in P(B)$.
   (c) $\{x\} \subseteq A - B$.
   (d) $|\, P(A) \,| = |\, P(B) \,|$.
   (e) more than one answer above is true.

6. $f : \mathbf{R} \rightarrow \mathbf{R}$ where $f(x) = \lfloor x/2 \rfloor$ is
   (a) a function that is one to one but not onto.
   (b) a function that is onto but not one to one.
   (c) a function that is one to one and onto.
   (d) a function that is neither one to one nor onto.
   (e) not a function.

7. $\bigcap_{i=1}^{\infty}\left[-1-\frac{1}{i},1+\frac{1}{i}\right]=$

(a) $(-1,1)$.
(b) $[-1,1]$.
(c) $(-2,2)$.
(d) Φ.
(e) none of the above.

In questions 8 and 9 below, suppose $g : A \to B$ and $f : B \to C$ are functions where

$$A = \{1,2,3,4\},\ B = \{a,b,c\},\ C = \{2,8,10\},$$

and g and f are defined by

$$g = \{(1,b),(2,a),(3,b),(4,a)\} \text{ and } f = \{(a,8),(b,10),(c,2)\}.$$

8. $f \circ g =$

(a) $\{(1,10),(2,8),(3,10),(4,8)\}$.
(b) $\{8,10\}$.
(c) $\{1,2,3,4\}$.
(d) $\{(2,2),(8,8),(10,10)\}$.
(e) not well-defined.

9. $f \circ f^{-1} =$

(a) $\{(1,10),(2,8),(3,10),(4,8)\}$.
(b) $\{2, 8,10\}$.
(c) $\{(a,a).(b,b),(c,c)\}$.
(d) $\{(2,2),(8,8),(10,10)\}$.
(e) not well-defined.

10. $\sum_{i=1}^{n}\sum_{j=1}^{i}(ij)=$

(a) $\sum_{i=1}^{n} i^2$.
(b) $\sum_{i=1}^{n} i^3$.
(c) $\sum_{i=1}^{n} \frac{i^3+i^2}{2}$.
(d) $\sum_{i=1}^{n} i^4$.
(e) none of the above.

Some Useful Formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad , \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad , \quad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1} \text{ where } a \neq 1 \quad , \quad \sum_{i=0}^{\infty} a^i = \frac{1}{1-a} \text{ where } |a| < 1,$$

$$\sum_{i=1}^{\infty} i a^{i-1} = \frac{1}{(1-a)^2} \text{ where } |a| < 1$$

$p \to (p \vee q)$	Addition	$[\neg q \wedge (p \to q)] \to \neg p$	Modus Tollens
$(p \wedge q) \to p$	Simplification	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$((p) \wedge (q)) \to (p \wedge q)$	Conjunction	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
$[p \wedge (p \to q)] \to q$	Modus Ponens	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws