Course No: CVL 2303
Course Title: Linear Algebra
Date: 20/11/2014
No. of Questions: (4 + Bonus)
Time: 90 Minutes
Using Calculator (No)

University of Palestine

Midterm Exam
2014/2015
Total Grade: 20 + 1⁺

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Student No.: _____
Student Name: _____
College Name: Civil Engineering
Dep. / Specialist: _____
Using Dictionary (Yes)

QUESTION ONE:

a) Given

$$\mathbf{A}=\begin{bmatrix}1 & 2 & 1\\ 1 & 1 & 2\\ 1 & 0 & 2\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}1 & 2 & 3\\ 3 & 1 & 2\\ 0 & 1 & 2\end{bmatrix}$$

Find: $[\mathbf{A}+\mathbf{B}+\mathbf{AB}]$ **1.0 points**

Find: $\det([\mathbf{A}+\mathbf{B}-\mathbf{AB}])$ **1.5 points**

b) Given

$$\mathbf{A}=\begin{bmatrix}1 & 0\\ 1 & 3\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}1 & 1\\ 2 & 5\end{bmatrix}$$

Find: $[\mathbf{A}+\mathbf{B}-\mathbf{AB}]^{-1}$ **1.5 points**

Verify: $\det(\mathbf{AB})=\det(\mathbf{BA})$ **1.0 points**

QUESTION TWO:

Solve the following linear system by either Gauss or Gauss-Jordan elimination procedure.

$$\begin{aligned} x_1 + 3x_2 + 5x_3 &= 1\\ -x_1 - 2x_2 &= 0\\ 2x_1 + 5x_2 + 4x_3 &= -1 \end{aligned}$$

4.0 points

QUESTION THREE:

a) Consider the given matrix equation

$$\begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \mathbf{X} = \begin{bmatrix} 2 & 1 \\ 0 & 0 \\ 1 & 3 \end{bmatrix}$$

1. **Find** the inverse of the most left matrix above using the adjoint

3.0 points

2. **Solve** the system for **X** using the matrix inversion procedure,, *the inverse is found in part **1***.

2.0 points

b) Find the inverse of the matrix using row operations:

$$\begin{bmatrix} \sigma & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sigma & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For what values of **σ** the above matrix is not invertible? ***2.0 points***

QUESTION FOUR:

Considering the system of the two algebraic linear equations

$$\begin{aligned} 2\,\mathbf{y} &= \frac{8}{\lambda} - \lambda\,\mathbf{x} \\ 2\,\mathbf{x} + \mathbf{y} &= 1 \end{aligned}$$

1. Express the system in its Matrix Form **Ax=B**

0.5 points

2**.** Find the values of **λ** such that the system has exactly one solution (for each allowed **λ)** .

1.0 points

3. Solve for **x** and **y**, generally in terms of the allowed **λ** in 2, using crammer's method, and check, by substitution, your solution for **x** and **y** for **λ** = 1. ***1.5 points***

4**.** From part 2 above, determine the values of **λ** for which the system has infinite solutions and for which the system has no solution. ***1.0 points***

BONUS QUESTION:

Simplify $\left(\mathbf{A}\,\mathbf{C}^{-1}\right)\left(\mathbf{A}\left(\mathbf{D}^{\mathbf{T}}\right)^{-1}\mathbf{C}^{-1}\right)^{-1}\left(\mathbf{A}\left(\mathbf{D}^{-1}\right)^{\mathbf{T}}\mathbf{C}^{-1}\right)\left(\mathbf{A}\,\mathbf{C}^{-1}\right)^{-1}$

1.0 points

End of Questions
Good Luck